Estimation of Satellite Lifetime from Orbital Failure Experience

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A method is proposed for estimating satellite lifetime from orbital failure data. The idea is to use orbital failure experience on all satellite subsystems together to estimate two average subsystem failure rates which are then used, along with the system reliability model and a priori piece part failure rates, to develop the estimate of system life. The subsystem reliability models are based on two constant failure rates, the first applying to a short period following launch during which a large portion of failures has been observed, and the second to the remainder of the system life. The development begins with the exponential reliability model, extends the results to a more general reliability model employing exponential subsystems, and then proceeds to the two failure rate model. In each case, confidence limits and probability density functions for the system mean life are presented. The method is then incorporated into a previously developed Monte-Carlo procedure for system replenishment analysis. Typical results are presented.

Introduction

ESTIMATES of satellite lifetimes are of great importance for establishing production schedules of current satellites and for determining the appropriate time to acquire follow-on systems. It is also important to know the accuracy of the estimate of satellite life; it is then possible to make probabilistic statements about the need for a new satellite on the date it becomes available, or to establish a production schedule so that there is a given probability (say 0.9) that a satellite will be available when needed.

The common procedure for estimating satellite lifetime is to use failure experience on individual parts to estimate the part failure rates (assumed constant with time), and then to use a reliability model of the system to transform these rates into a system reliability function. This approach has the advantage of using the large quantities of failure data which are usually available on piece-parts and is useful for predicting the adequacy of a design to meet lifetime specifications. However, the method is not convenient for updating production schedules based on orbital failure experience. Also, since each lifetime estimate requires new piece-part reliability evaluations, the time to produce new estimates is inconveniently long.

A second method is based on a very simple reliability model (usually an exponential system reliability function) for the entire system, and uses system (i.e., satellite) failure data to estimate the system failure rates. The result is a simple procedure for estimating satellite lifetimes, but since there is not much data (typically one or two failures), the confidence limits on the estimate are broad, i.e., the variance in the estimate of satellite lifetime is large. Furthermore, the exponential model is overly simplified and does not adequately represent the system.

The method proposed here introduces two new concepts. First, two failure rates are used, a high rate applying to the initial "burn-in" period and a much lower rate applying subsequently. Second, orbital failure experience on all subsystems together is used to estimate the two average subsystem failure rates, which are then used with the system reliability model and a priori piece-part failure rates to develop an improved estimate of system life, along with the corresponding confidence limits.

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Exponential Reliability Model

The simplest and most conveniently analyzed case is that of failures occurring randomly in time (Poisson distributed) with constant average failure rate λ, which leads to a reliability function

$$R(t)=e^{-\lambda t} \tag{1}$$

The mean life is then

$$\theta=\int_0^{t_c} R(t)\mathrm{d}t=\frac{1}{\lambda}(1-e^{-\lambda t_c}) \tag{2}$$

where t_c is the truncation time—the time beyond which it is assumed impossible for the system to survive due to wearout or depletion. When $t_c=\infty$, the mean life is simply the reciprocal of the failure rate. The exponential model is usually used to represent the behavior of parts, or occasionally subsystems, but the redundancy designed into a complete satellite system makes the constant failure rate assumption less satisfactory in that case. Nevertheless, this model has been used to obtain crude estimates of satellite life from satellite failure experience; the method is also the basis of the subsequent development and will, therefore, be described here in some detail.

The objective is to estimate the failure rate λ and the mean life θ from observations of the number of failures r that have occurred in time t. For constant failure rate, the probability density of failure time is exponential

$$f_t(t)=\mathrm{d}R/\mathrm{d}t=\lambda e^{-\lambda t} \tag{3}$$

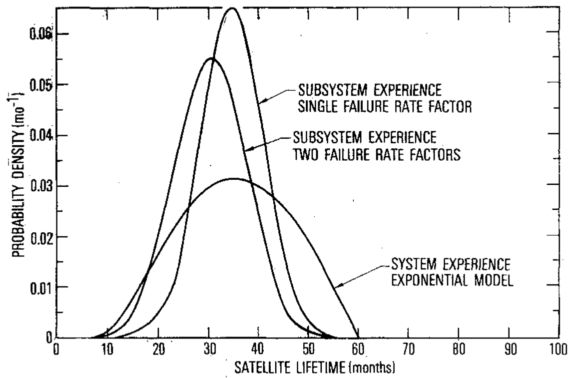

Fig. 1 Density of MMD for various methods of estimation (60-month truncation).

and the density of the number of failures r occurring in time t is Poisson

$$f_r(r) = e^{-\lambda t}(\lambda t)^r/r! = e^{-m}m^r/r! \tag{4}$$

where, for convenience, $m = \lambda t$.

Equation (4) gives the probability of r failures, given the value of m; but it is desired to know the probability density function for m, given r failures. It follows from Bayes Law,[1] that the density of m, given r failures in time t, is given by the same expression

$$f(m) = e^{-m}m^r/r! \tag{5}$$

In terms of r, this was the Poisson density; in terms of m, it is the gamma density. The gamma distribution is, then

$$\gamma(m) = \int_0^m \frac{e^{-u}u^r}{r!}\,\mathrm{d}u$$

$$= 1 - e^{-m}\left[1 + m + \frac{m^2}{2!} + \ldots + \frac{m^r}{r!}\right] \tag{6}$$

Equations (5) and (6) provide the density of m (i.e., the failure rate) and the distribution, which gives the upper confidence limit on m. The distribution can also be expressed in terms of the χ^2 distribution, for which tables are more readily available.

$$\gamma(m) = \int_0^m \frac{e^{-u}u^r}{r!}\,\mathrm{d}u = \int_0^{2m} \frac{e^{-k/2}w^{(k/2)-1}}{[(k/2)-1]!2^{k/2}}\,\mathrm{d}w \tag{7}$$

Tables of the χ^2 distribution give the value of $2m$ as a function of k and γ (or $1-\gamma$).

Also of interest are the distribution and density of the mean life θ. The probability of θ exceeding a particular value is given by $\gamma(m(\theta))$, where $m(\theta)$ is obtained from Eq. (2), more conveniently written

$$\theta(m) = (t/m)\,[1 - \exp(-mt_c/t)] \tag{8}$$

The density of θ is

$$f_\theta(\theta) = -\frac{\mathrm{d}\gamma}{\mathrm{d}\theta} = -\frac{\mathrm{d}\gamma}{\mathrm{d}m} \Big/ \frac{\mathrm{d}\theta}{\mathrm{d}m} = -f(m) \Big/ \frac{\mathrm{d}\theta}{\mathrm{d}m} \tag{9}$$

where the derivative is obtained by differentiating Eq. (8). [Figure 1 contains a plot of $f_\theta(\theta)$ for a 60-month truncation time and 78 months accumulated life with one failure. This figure also contains density functions for the more appropriate reliability models developed next.]

More Complete Reliability Model

The exponential model for system reliability provides gross estimates of failure rate and failure time, but the paucity of data for most space systems (e.g., one failure in 78 months) results in a large uncertainty in the estimates. Considerably more failure data are available on subsystems, e.g., in the above case 12 failures in 111 months of operation. Because of redundancy, subsystem failures do not necessarily result in satellite failure. The subsystem redundancy incorporated in the satellite considered here is illustrated in Fig. 2, and a list of subsystem failures is given in Table 1. The numbers above each box are estimates of subsystem failure rates† based on piece-part test data and orbital experience on similar subsystems on other spacecraft. The subsystem failure rates are assumed constant, resulting in exponential subsystem reliability functions. The system reliability function, obtained by appropriately combining the redundant units and based on these subsystem failure rates, is shown in Fig. 3 as $R_0(t)$.‡ Appendix A shows the analytic form of $R_0(t)$. The essence of the method proposed here is to use the subsystem failure data to estimate a scale factor k such that $R(t) = R_0(kt)$. When burn-in is taken into consideration, two such rates are determined, as will be described subsequently.

†Units are failures per 10^9 hr.

Table 1 Box failures

Flight	Box	λ	Redundancy[a]
A	L2 XMTR	5005	SB[b]
	L3 RCVR	4884	AR[b]
	CEA	19722	SB[b]
	BATT	6924	TT
	DPES TMU	9419	SB
B	L2 XMTR	5005	SB[b]
	L3 DCD	7142	AR
C	L3 RCVR	4884	AR[b]
D	VDA	1294	SB
	L2 XMTR	5005	SB
	SSA	13	AR
	DPES DCS	8492	SB

[a]Failure occured during first 60 days. [b]SB: standby redundant, AR: active redundant, TT: two out of three required.

If it is now assumed that the relative values between the a priori failure rates are unchanged, the estimates of the individual subsystem failure rates become

$$\hat{\lambda}_i = (\hat{\lambda}_T/\lambda_{T0})\lambda_i = k\lambda_i \tag{10}$$

where $k = \hat{\lambda}_T/\lambda_{T0}$. When these subsystem failure rates are put together to form a system reliability function, the constant k appears as a scale factor on t, i.e., everywhere that t appears, it is in the form $\lambda_i t$, and each λ_i has the scale factor k attached; hence, the estimated reliability function is

$$R(t) = R_0(kt) \tag{11}$$

with k given by Eq. (10). For the case of $r = 12$ and $t = 111$ months and the model shown in Fig. 2, λ_{T0} turns out to be 0.13217 month^{-1} and $\hat{\lambda}_T = 0.12294$ month^{-1} (60% confidence), resulting in $k = 0.93017$. So, the reliability function is slightly above $R_0(t)$ in Fig. 3. For comparison, the exponential reliability function based on the 60% confidence failure rate computed by the method used in the previous section is also shown. For the example considered here, accumulated time on satellites was $t = 87$ months with 1 failure, compared to 111 months on subsystems with 12 failures.

The density and distribution of the mean life θ are computed as in the previous section, except that $\theta(m)$ is obtained by numerical integration of the reliability function $R(t) = R_0(kt)$, with

$$k = \hat{\lambda}_t/\lambda_{T0} = m/(\lambda_{T0}t) \tag{12}$$

where the variable t in this equation represents the system operational time. It is not the time variable appearing in $R(t)$. Thus, the idea is to use the a priori reliability model to describe the failure rates of one unit relative to another, but to use the on-orbit subsystem failure rates to estimate an average system failure rate.

Data are obtained on all the units that are active, so conceptually a system is constructed consisting of all active subsystems in series. The total a priori failure rate of the conceptual system (failure rates sum for exponential model in series) is

‡The figure also contains the exponential reliability function and the 60% confidence reliability functions for one and two failure rates developed in this and the following sections.

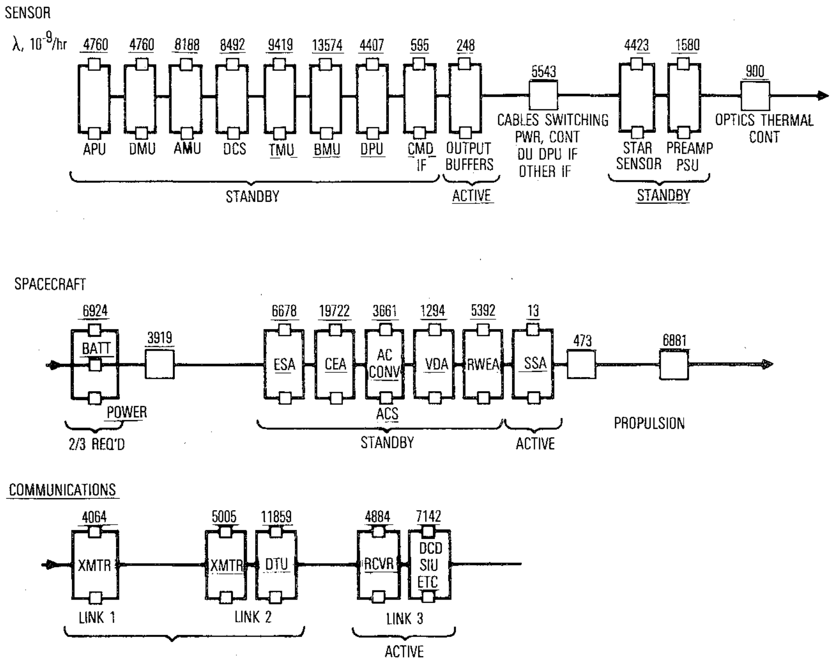

Fig. 2 Satellite reliability model.

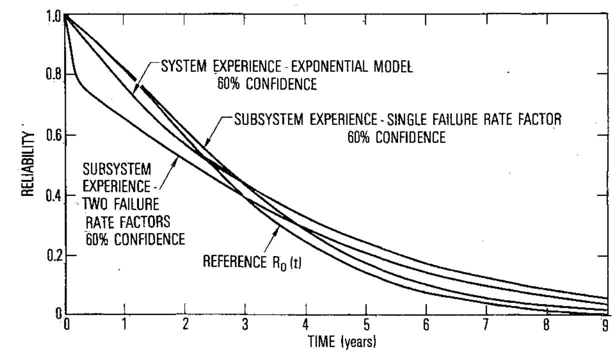

Fig. 3 Comparison of reliability functions.

$$\lambda_{T0} = \sum_i \lambda_{si} + 2\sum_i \lambda_{api} + \sum_i \lambda_{spi} + 2\lambda_{tt} \quad (13)$$

where

λ_{si} = a priori rates of series elements

λ_{api} = rates of active parallel elements

λ_{spi} = rates of standby parallel elements

λ_{tt} = rate of element requiring two out of three for success

If r failures have been observed in t months of system operation, the density of the estimate $\hat{\lambda}_T$ of the total failure rate λ_T is obtained from Eq. (5) with $m = \hat{\lambda}_T t$§.

Figure 1 shows the density $f_\theta(\theta)$ for $r = 12$ and $t = 111$ months when the truncation time t_c is 60 months. Also shown are the corresponding densities for the method described in the previous section, using the exponential reliability function based on one failure in 87 months of system operation. The other densities shown in the figure are based on the assumption of two failure rates, as described in the following section. Immediately apparent from the figure is the increased sharpness of the densities based on subsystem failure experience.

§Time on failed units should, strictly speaking, be substrated from t, but the effect is small and has been neglected, i.e., replacement has been assumed.

Two Failure Rate Model

Failure data show a marked tendency to fall into two categories, with nearly half of all failures having occurred during the first two months of on-orbit operation. Figure 4 shows the breakdown of all system discrepancies for the system considered here.¶ This suggests the possibility of generalizing the constant failure rate model (of subsystems) to one employing two constant failure rates: the first applying for a period t_I of about two months after launch,** and the second rate applying for the duration of the subsystem life.

For the ith subsystem with a priori failure rate λ_i, the a priori reliability function is

$$R_{0i}(t) = \exp^{(-\lambda_i t)} \quad (14)$$

If, as in Eq. (10), it is assumed that the a priori failure rates are multiplied by constants (k_I for $t \le t_I$ and k_2 for $t > t_I$), then the subsystem reliability function becomes

$$\begin{aligned} R_i(t) &= \exp(-k_I \lambda_i t) && t \le t_I \\ &= \exp[-k_I \lambda_i t_I - k_2 \lambda_i (t - t_I)] && t > t_I \end{aligned} \quad (15)$$

where $R_i(t/t_I)$ is the probability of surviving to time t, given survival to time t_I.

In terms of the a priori reliability function $R_{0i}(t)$, Eq. (15) can be written

$$\begin{aligned} R_i(t) &= R_{0i}(k_I t) && t \le t_I \\ &= R_{0i}(k_I t_I + k_2(t - t_I)) && t > t_I \end{aligned} \quad (16)$$

which, for subsystems, is analogous to Eq. (11). But, because of the standby redundant elements, the analogy does not carry over to the system reliability function and the development is somewhat complicated. The appropriate expressions are developed in Appendix B.

With the form of the reliability function determined, the constants k_I and k_2 are estimated as in the previous section. If

¶The figure includes anomalies not classified as failures. Only failures are used in the model developed here; the figure is shown only to illustrate the phenomenon of early failures.

**Or, in the case of standby redundant elements, for a period t_I after the unit is turned on.

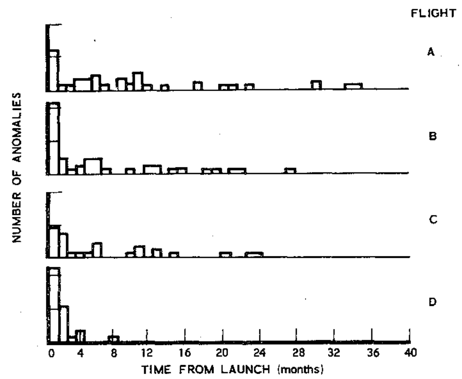

Fig. 4 Discrepancy experience.

r_1 failures have occurred during t_{11} months of system operation in the period $t \leq t_1$, then the density of the estimate $\hat{\lambda}_{T1}$ of the total burn-in failure rate λ_{T1} is obtained from Eq. (5) with $r = r_1$ and $m = \hat{\lambda}_{T1} t_{11}$. Similarly, for r_2 failures during t_{22} months of system operation in the period $t > t_1$, Eq. (5) is used with $r = r_2$ and $m = \lambda_{T2} t_{22}$. Then, by appropriately ratioing the subsystem failure rates, we obtain as in Eq. (10)

$$\hat{\lambda}_{i1} = (\hat{\lambda}_{T1} / \lambda_{T0}) \lambda_i = k_1 \lambda_i \tag{17a}$$

$$\hat{\lambda}_{i2} = (\lambda_{T2} / \lambda_{T0}) \lambda_i = k_2 \lambda_i \tag{17b}$$

where

$$k_1 = \hat{\lambda}_{T1} / \lambda_{T0} \quad k_2 = \hat{\lambda}_{T2} / \lambda_{T0} \tag{18}$$

and λ_{T0} is the a priori total failure rate given by Eq. (13).

The 12 subsystem failures in 111 months total subsystem experience can be divided into 5 in 8 months experience prior to $t_1 = 2$ months, and 7 in 103 months experience subsequent to t_1. For these values, and the model of Fig. 2, λ_{T0} is 0.13217 months^{-1} and the upper 60% confidence limits for $\hat{\lambda}_{T1}$ and $\hat{\lambda}_{T2}$ turn out to be 0.78366 and 0.08133 months^{-1} with $k_1 = 5.929$ and $k_2 = 0.61534$. The reliability function based on these rates is included in Fig. 3, along with those described previously.

Computation of the distribution and density of the mean life θ given the subsystem failure data is more complicated in this case because θ now depends on two random variables, k_1 andk_2, which are each gamma distributed according to Eqs. (5) and (6), with

$$m_1 = \lambda_{T1} t_{11} = k_1 \lambda_{T0} t_{11} \tag{19a}$$

$$m_2 = \lambda_{T2} t_{22} = k_2 \lambda_{T0} t_{22} \tag{19b}$$

So, the joint density must be integrated over the appropriate region of the m_1, m_2 space. A computer program has been developed to perform the integration. The results for the case of $r_1 = 5$, $t_{11} = 8$ months, $r_2 = 7$, $t_{22} = 103$ months, $t_1 = 2$ months are shown in Fig. 1 for $t_c = 60$ months. The densities are somewhat broader than in the single rate case, since the power of the data is diluted, due to the need to estimate two parameters rather than one. However, the two rate model more accurately reflects the actual situation, as evidenced by the failure experience and is, therefore, recommended for satellite availability calculations, as will be discussed in the following section.

Various estimates of mean life can be made, based on the densities shown in Fig. 1. Two of these are shown in Table 2. Although there is a relatively small variation between the different models, it should not be concluded that there is no effect on subsequent launch scheduling. Once a satellite has survived the burn-in period, the failure rate will be much lower and the mean life much higher than when the single rate model is used. For example, using the previous failure data and 60% confidence failure rates, the mean remaining life of a satellite which has been two months in orbit without a subsystem failure becomes 37.8 months, compared to 32.7 months, using the single rate model.

Table 2 Mean life estimates using various models and estimators (60-month truncation)

Estimate	Mean	60% confidence
System experience, exponential model	35.5	32.3
Subsystem experience, single failure rate factor	34.7	33.1
Subsystem experience, two failure rate factors	30.4	28.8

System Replenishment Analysis

Present Generalized Availability Program

At The Aerospace Corp., system replenishment analysis is being done by means of the generalized availability program†† (GAP).[2,3] The program uses input reliability functions to generate random failure times for each satellite now on orbit. Then, given a production schedule and assuming launch of a new satellite at a fixed time after failure of an existing one (or when the satellite is produced, if that is later), failure times are generated for the replacement satellites and the process continues, yielding launch times and failure times for however many satellites are input into the program. This process is repeated a number of times (usually 1000), and the failure data are assembled into histograms indicating the probability that each of the satellites will be needed on any given date.

In previous versions of GAP the input reliability function has been chosen to yield the same mean life as the exponential system model with a 60% confidence limit, i.e., $\gamma[m(\theta)]$ in Eq. (6) = 0.6 with $\theta(m)$ given by Eq. (8). However, the form of the reliability function used in GAP was not exponential, but was a Weibull function

$$R(t) = e^{-(t/\alpha)^{\beta}} \tag{20}$$

with α chosen to give the correct mean life, and the shape parameter β being based on fits to a predetermined reliability model, such as is shown in Fig. 2.

Randomized Failure Rates

Two improvements to the system replenishment analysis have been implemented in GAP as version D. First, the actual two rate reliability function $R(t)$ is computed for the model in Fig. 2, as described in Appendices A and B. Second, the two failure rate factors k_1 and k_2 are computed from m_1 and m_2 using Eq. (19), where m_1 and m_2 are generated randomly on each Monte Carlo cycle from the gamma distribution Eq. (6)‡‡ with $r = r_1$ and r_2, respectively. This procedure accounts for the uncertainty in the reliability function due to the limited amount of failure data and makes it unnecessary to choose an input failure rate corresponding to a mean life at a particular confidence level.§§

The procedure implemented is to generate k_1 and k_2 and then compute the reliability function at the points t_0, t_1, and $t_0 + t_1$, which define boundaries between different analytical expressions for the reliability function (see Appendix B), and

††GAP program was developed by S. Sugihara and H. Wong, who have also implemented the modifications described in this paper.

‡‡The simulation uses Weibull fits to $\gamma(m)$ for various values of r, since the Weibull function is more easily solved for m, given γ, which is generated randomly on the interval (0,1) on each Monte Carlo cycle.

§§The idea of randomizing the reliability function, but in a different way, was first suggested by Sugihara.[4]

at two points t_2 and t_3 at 30 and 60 months. Then, the reliability R is generated, uniform in the interval (0, 1). The problem is then to solve for the failure time t_1 given $R(t)$. This is done by using a linear approximation to $R(t)$ in the intervals $(0, t_0)$, (t_0, t_1) and (t_1, t_0+t_1), and a Weibull fit through the points at t_0+t_1, t_2, t_3 in the interval (t_0+t_1, t_c).

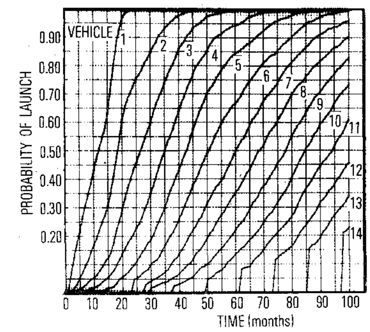

Fig. 6 Probability of launch for random truncation and 60% confidence failure rate factors.

Randomized Truncation Time

Abrupt truncation of satellite life is commonly assumed but is difficult to justify on physical grounds, since the wearout phenomena which the assumption represents are themselves random or are otherwise difficult to predict. Analysis of the various wearout and depletion effects (bearings, fuel depletion, solar cell degradation, detector thermal noise, etc.) leads to the belief that wearout is very unlikely to occur before four years, is most likely to occur around five years, and could occur as late as seven years. A probability density function with these properties is the Rayleigh

$$f_c(t_c) = 0 \tag{21a}$$

$$f_c(t_c) = [(t_c - t_0)/\alpha^2]\exp[-(t_c - t_0)^2/2\alpha^2] \quad t_c \geq t_0 \tag{21b}$$

The distribution function corresponding to the Rayleigh density is Gaussian, which is convenient for Monte Carlo simulation

$$F_c(t_c) = \int_{t_0}^{t_c} f_c(x)\,dx = 1 - \exp[-(t_c - t_0)^2/2\alpha^2] \tag{22}$$

The mean of the distribution is $t_0 + \alpha(\pi/2,)^{1/2}$ so for $t_0 = 48$ months and a mean of 60 months, $\alpha = 12(2/\pi)^{1/2} = 9.575$ months. The probability of truncation occurring before six years is 0.957, and before seven years is 0.999.

The program includes the capability to select t_c randomly on each Monte Carlo cycle from the Rayleigh distribution.

Results

Figure 5 presents the results of a GAP run using the randomized reliability function and randomized truncation time for a supply of three satellites. For comparison, Fig. 6 shows the same case based on a fixed reliability function using 60% confidence failure rates. In the example, satellites 2, 3, and 4 are already in orbit, with truncation times of 13, 35, and 50 months, respectively. In addition, their reliability functions were modified to account for the failures listed on Fig. 2. The reliability functions for those satellites, based on 60% confidence failure rates, are shown in Fig. 7.

When only one satellite is considered, the histogram of failure times yields a mean reliability function shown in Fig. 7. Compared with the reliability function based on 60% confidence failure rates, the randomized case shows fewer failures for times beyond two years. The mean life corresponding to the mean of the randomized reliability function is 30.4 months, compared to 28.8 months when 60% failure rates are used.

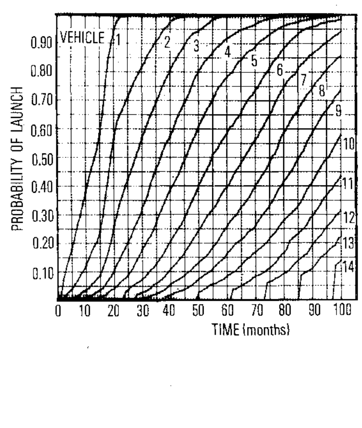

Fig. 6 Probability of launch for random trunction and 60% confidence failure rate factors.

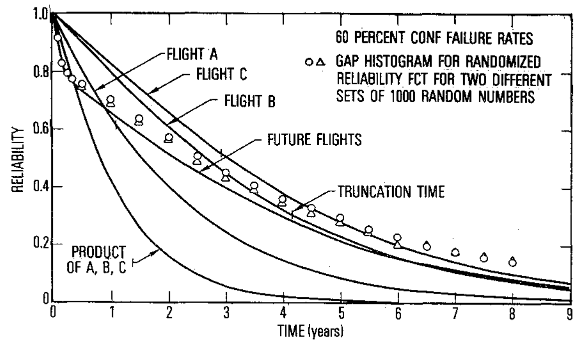

Fig. 7 Reliability functions for $t = 8, 103$ and $r = 5.7$.

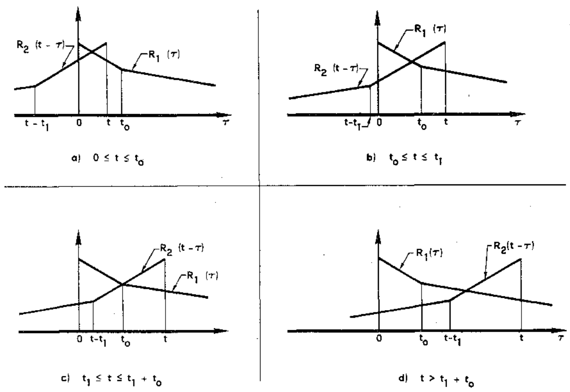

Fig. 8 Two-rate reliability function.

Conclusions

A method has been developed for using on-orbit failure experience on subsystems (or modules) to develop improved estimates of satellite lifetime. The observed tendency of failures to occur shortly after launch has been incorporated into the method by means of a two-rate reliability model. Statistical significance is improved by using all subsystem failures to determine the overall system lifetime. Ratios between subsystem failure rates continue to be established by the a priori reliability model.

The ultimate use for the method is to establish satellite procurement schedules. The procedure for utilizing this approach for satellite replenishment analysis has been described and typical results presented. This approach to replenishment analysis is currently being used on at least one operational program. The essential benefit of the approach is that it provides a rational way to incorporate limited failure data into a complex reliability model, thereby increasing confidence in the results.

It should be noted that no consideration has been given to satellite shelf life. It is apparent that there are many mechanisms which cause a degradation of reliability while the satellite is being stored prior to launch, and it would be a straightforward modification to model this effect in the GAP program. However, the effect of shelf life does appear in on-orbit failure data used to estimate the system lifetime; hence, the statistical effect of shelf life is included implicitly in the results.

Appendix A: Reference Reliability Function

The reliability model shown in Fig. 2 contains series elements plus three kinds of redundant combinations: 1) active parallel, 2) standby parallel, and 3) two-out-of-three parallel. The reliability functions for these combinations are easily derived and are available in various reliability textbooks.[5] The subsystems are assumed to have exponential reliability functions, with failure rates as denoted in Fig. 2. The reliability functions of the combinations are

Series:

$$\exp(-\sum_i \lambda_{si} t)$$

Active parallel:

$$\prod_i [2\exp(-\lambda_{api}t) - \exp(-2\lambda_{api}t)]$$

Standby parallel:

$$\prod_i [(1 + \lambda_{spi}t)\exp(-\lambda_{spi}t)]$$

Two-our-of-three parallel:

$$3\exp(-2\lambda_{tt}t) - 2\exp(-3\lambda_{tt}t)$$

where the sum and products are taken over the elements of that type. For the model of Fig. 2, there are four series elements, four active parallel combinations, and 18 standby parallel combinations. Note that the expression for standby parallel redundancy assumes no degradation of the standby element. However, comparison with a reliability function, assuming $\lambda = 0.1\lambda_{spi}$ while on standby, shows an insignificant effect on the results.

The reference reliability function $R_0(t)$ is the product of the four expressions just given.

Appendix B: Reliability Function with Two Failure Rates

General Expression for Standby Redundancy

Based on one failure rate, survival to time t can occur two ways: a) the primary element survives to time t, or b) the primary element fails at time $\tau < t$ and the standby element survives from τ to t. The probabilities of the two options are

$$P(A) = R_1(t) \tag{B1a}$$

$$P(B) = \int_0^t f_1(\tau) R_2(t-\tau)\,d\tau \tag{B1b}$$

where $R_1(t)$ and $R_2(t)$ are the reliability functions of the primary and redundant elements, and $f_1(t)$ is the density of failure times of the primary element

$$f_1(t) = dR_1(t)/dt \tag{B2}$$

In Eq. (B1b), $f_1(\tau)d\tau$ is the probability of failure of the primary unit between τ and $\tau + d\tau$, and $R_2(t-\tau)$ is the probability of the standby unit surviving from τ to t. $P(B)$ is the integral over all possible failure times of the primary element. The probability of the redundant combination surviving to time t is then

$$R(t) = P(A) + P(B) \tag{B3}$$

Two Rate Model

The failure rate λ_1 is assumed to apply to each element for a time t_1 after the element is turned on, and the rate λ_2 thereafter, resulting in the subsystem reliability function given by Eq. (15). However, Eq. (15) assumes t is measured from time of launch; to account for satellites already in orbit, it is necessary to introduce the remaining time t_0 $(0 \le t_0 \le t_1)$, during which the primary elements exhibit the failure rate λ_1. For satellites which have been in orbit longer than t_1, $t_0 = 0$; for satellites not yet launched, $t_0 = t_1$. So, the reliability functions in Eqs. (B1) are different.

$$R_1(t) = \exp(-\lambda_1 t) \qquad t \le t_0 \tag{B4a}$$

$$= \exp(-\lambda_1 t_0 - \lambda_2(t-t_0)) \qquad t > t_0 \tag{B4b}$$

$$R_2(t-\tau) = \exp[-\lambda_1(t-\tau)] \qquad t-\tau \le t_1 \tag{B5a}$$

$$= \exp[-\lambda_1 t_1 - \lambda_2(t-\tau-t_1)] \qquad t-\tau > t_1 \tag{B5b}$$

The inequalities in Eqs. (B4) and (B5) lead to different expressions for the integrals in Eqs. (B1) for the four time regions, $t \le t_0$, $t_0 \le t \le t_1$, $t_1 \le t \le t_1 + t_0$, $t \ge t_1 + t_0$. Figure 8 shows the functions $R_1(\tau)$, and $R_2(t-\tau)$ for the four cases [$f_1(\tau)$ is computed from $R_1(\tau)$ using Eq. (B2)]. With the help of the figure, the integrand for $P(B)$ in Eq. (B1b) can be evaluated and the integrations carried out, leading to the reliability function of the redundant combination

$$R(t) = (1 + \lambda_1 t)\exp{-\lambda_1 t} \qquad t \le t_0 \tag{B6a}$$

$$R(t) = \frac{\lambda_1}{\lambda_1 - \lambda_2}\exp(-\lambda_1 t_0 - \lambda_2(t-t_0)) + \left[\lambda_1 t_0 - \frac{\lambda_2}{\lambda_1 - \lambda_2}\right]\exp(-\lambda_1 t) \qquad t_0 \le t \le t_1 \tag{B6b}$$

$$R(t) = \frac{\lambda_1}{\lambda_1 - \lambda_2}\{\exp[-\lambda_1 t_0 - \lambda_2(t-t_0)] + \exp[-\lambda_1 t_1 - \lambda_2(t-t_1)]\} + \left[\lambda_1(t_0 + t_1 - t) - \frac{\lambda_1 + \lambda_2}{\lambda_1 - \lambda_2}\right]\exp(-\lambda_1 t) \qquad t_1 \le t \le t_1 + t_0 \tag{B6c}$$

$$R(t) = \frac{\lambda_1}{\lambda_1 - \lambda_2}\{\exp[-\lambda_1 t_0 - \lambda_2(t-t_0)] + \exp(-\lambda_1 t_1 - \lambda_2(t-t_1))]\} + \left[\lambda_2(t - t_1 - t_0) - \frac{\lambda_1 + \lambda_2}{\lambda_1 - \lambda_2}\right]\exp[-\lambda_1(t_1 + t_0) - \lambda_2(t - t_1 - t_0)] \qquad t \ge t_1 + t_0 \tag{B6d}$$

Computation of System Reliability Function

The system reliability function is computed in two parts: $R_1(t)$, which contains all but the standby parallel elements, and $R_2(t)$, which contains only the standby parallel elements. $R_1(t)$ is computed from the reference reliability function R_{10} (excluding standby parallel elements), as described in Appendix A. R_{10} is used with different arguments as follows:

$$R_1(t) = R_{10}(k_1 t) \qquad t \le t_1 \tag{B7a}$$

$$R_i(t) = R_{10}(k_1 t_1 + k_2(t-t_1)) \qquad t > t_1 \tag{B7b}$$

The reliability function for the standby elements is computed as the product

$$R_2(t) = \prod_i R_{2i}(t) \tag{B8}$$

where the R_{2i} are computed from Eqs. (B6) with

$$\lambda_1 = k_1 \lambda_{spi} \quad \lambda_2 = k_2 \lambda_{spi} \tag{B9}$$

where λ_{spi} are the a priori failure rates of the standby parallel elements. The system reliability function is then the product

$$R(t) = R_1(t)R_2(t) \quad \text{(B10)}$$

The constants k_1 and k_2 are generated randomly in GAP as described previously, based on subsystem failure experience. Alternatively, the k's can be selected at a particular confidence level, usually 60%. In either case, the m_i are selected from the gamma distribution [Eq. (6)] with $r = r_i (i = 1, 2)$.

$$\gamma(m_i) = 1 - e^{-m_i}\left[1 + m_i + \frac{m_i^2}{2!} + \ldots + \frac{m_i^{r_i}}{r_i!}\right] \quad \text{(B11)}$$

When the 60% confidence values are desired, $\gamma = 0.6$. In GAP, however, γ is chosen randomly, uniform on the interval (0, 1). Then, as described previously

$$k_i = m_i / (\lambda_{T0} t_{ii}) \quad i = 1, 2 \quad \text{(B12)}$$

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